Announcements

- HW 0 due in class (solutions posted later today).
- HW 1 posted today. Due next Friday (Apr 11).
- Lecture notes to be updated online after class.
- Labs start next week (Experiment 0).
  - Read lab beforehand
  - Each lab group will get a 331 Lab Kit
  - Bring probes, breadboard, components from 233 (will need to buy if lab group doesn't have enough)
- Read Chapter 2.

Extrinsic Semiconductors – Carrier Density (*n*-type)

- Silicon doped with donors with density N_D ($N_A = 0$)
  - Solve $np = n_i^2$ and $N_D + p - n = 0$
  - Eliminate p:

$$N_D + \frac{n_i^2}{n} - n = 0 \Longrightarrow n^2 - nN_D - n_i^2 = 0$$

  - Solve: $n = \frac{N_D + \sqrt{N_D^2 + 4n_i^2}}{2}, p = \frac{n_i^2}{n}$

Negative solution tossed out

  - Most of the time: $N_D \gg n_i$, thus $n \cong N_D, p \cong \frac{n_i^2}{N_D}$ $\Rightarrow$ $n \gg n_i \gg p$, the semiconductor is called "*n*-type"
  - Electrons: majority carrier; Holes: minority carrier

Extrinsic Semiconductors – Carrier Density (*p*-type)

- Silicon doped with acceptors with density N_A ($N_D = 0$)
  - Solve $np = n_i^2$ and $p - N_A - n = 0$
  - Eliminate n:

$$p - N_A - \frac{n_i^2}{p} = 0 \Longrightarrow p^2 - pN_A - n_i^2 = 0$$

Negative solution tossed out

  - Solve: $p = \frac{N_A + \sqrt{N_A^2 + 4n_i^2}}{2}, n = \frac{n_i^2}{p}$
  - Most of the time: $N_A \gg n_i$, thus $p \cong N_A, n \cong \frac{n_i^2}{N_A} \Rightarrow$ $p \gg n_i \gg n$, the semiconductor is called "*p*-type"
  - Holes: majority carrier; Electrons: minority carrier

Extrinsic Semiconductors – Carrier Density (compensation)

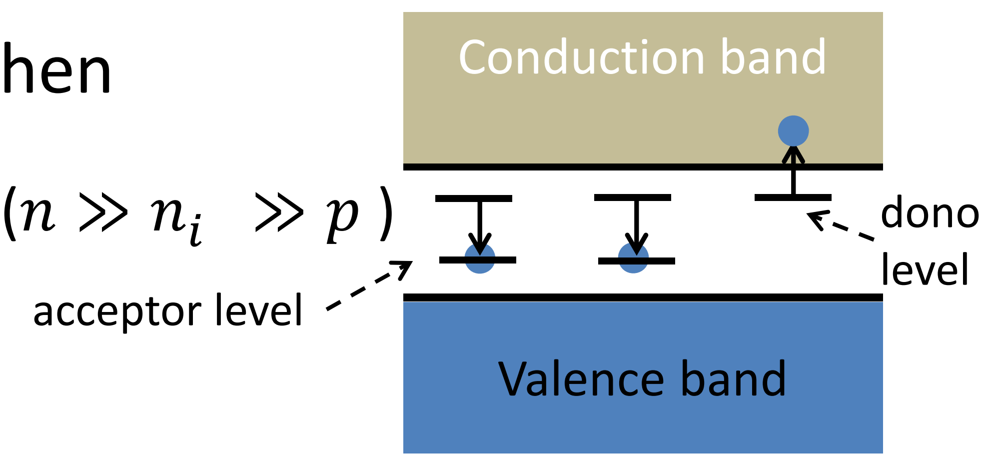
- Silicon doped with both donors (N_D) and acceptors (N_A)
  - Suppose $N_D > N_A$ (donors dominate)

$$n = \frac{(N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2} > n_i \; , \quad p = \frac{n_i^2}{n} < n_i \quad (n\text{-type})$$

  - If : $N_D - N_A \gg n_i$, then

$$n \cong N_D - N_A, p \cong \frac{n_i^2}{N_D - N_A} \; (n \gg n_i \gg p)$$

- Compensation



Compensated semicondutor

Extrinsic Semiconductors – Carrier Density (compensation)

- Silicon doped with both donors (N_D) and acceptors (N_A)
  - Suppose $N_D < N_A$ (acceptors dominate)

$$p = \frac{(N_A - N_D) + \sqrt{(N_A - N_D)^2 + 4n_i^2}}{2} > n_i \;, n = \frac{n_i^2}{p} < n_i \quad (p\text{-type})$$

  - If : $N_A - N_D \gg n_i$, then

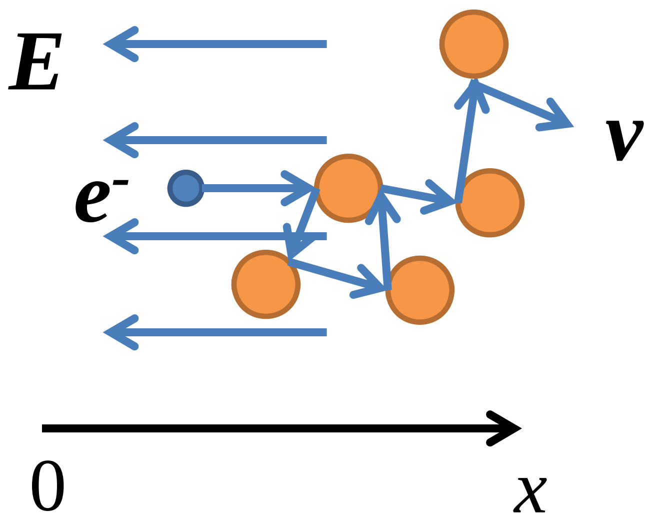
$$p \cong N_A - N_D, n \cong \frac{n_i^2}{N_A - N_D} \;(\, p \gg n_i \gg n \,)$$

Extrinsic Semiconductors – Carrier Density

- If the (net) dopant density is much larger than n_i , then the majority carrier density is almost equal to the dopant density.
- The minority carrier density can then be obtained by the mass action law $np = n_i^2$.

Recall: Electrons in Motion

Constant electric field in a solid



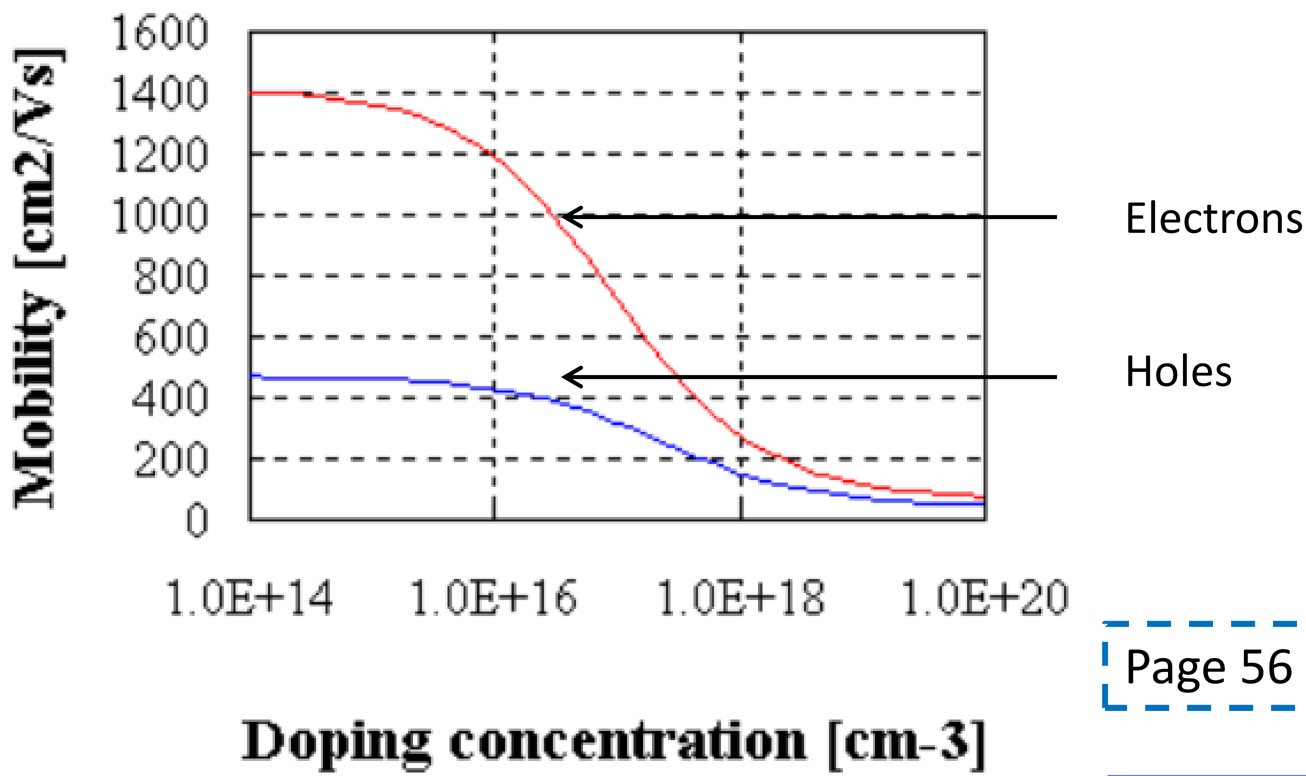
Electron flies for short intervals (~ps) before bumping into scattering objects

Result: Electron's average velocity proportional to field

$$v = \mu E$$

Extrinsic Semiconductors Mobility & Resistivity

- Impurities introduce scattering centers that degrade the mobility of both carriers (e^- & h^+).



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Extrinsic Semiconductors Mobility & Resistivity

- Doping affects resistivity via both mobility and density of carriers:

$$\sigma = q(n\mu_n + p\mu_p)$$

- Example: Si doped with As ($N_D = 10^{17}\mathrm{cm}^{-3}$), what is the conductivity at 300 K?

Step 1: find n and p :

At 300 K, $n_i = 1.0 \times 10^{10}\mathrm{cm}^{-3} \ll N_D$, so

$$n \cong N_D = 10^{17}\mathrm{cm}^{-3}$$

$$p \cong n_i^2/N_D = 1.0 \times 10^3\ \mathrm{cm}^{-3}$$

Extrinsic Semiconductors
Mobility & Resistivity

Step 2: find μ_n and μ_p:

From mobility graph, find for $N_D = 10^{17}$ cm^{-3}:

$\mu_n = 700$ cm^2/(V·s); $\mu_p = 260$ cm^2/(V·s)

Step 3: Find σ:

$$\sigma = q(\mu_n n + \mu_p p)$$

$\sigma = 1.6\times10^{-19}$ C (10^{17}cm$^{-3}\times700$cm^2/V-s + $1.0\text{x}10^3$cm$^{-3}\times260$ cm^2/V-s)

= 1.6×10^{-19} C (7×10^{19}cm^{-3} + ~~2.6×10^5~~) cm^{-1}/V-s

= 11.2 Ω^{-1}cm^{-1} (S/cm, ℧/cm)

Hole contribution is negligible
Electron contribution dominates

$\rho = 1 / \sigma = 0.089\ \Omega \cdot$ cm

Conduction Mechanisms

- Drift Current:

Electrons: $j_n^{\text{drift}} = -qnv_n = -qn(-\mu_n E) = qn\mu_n E$

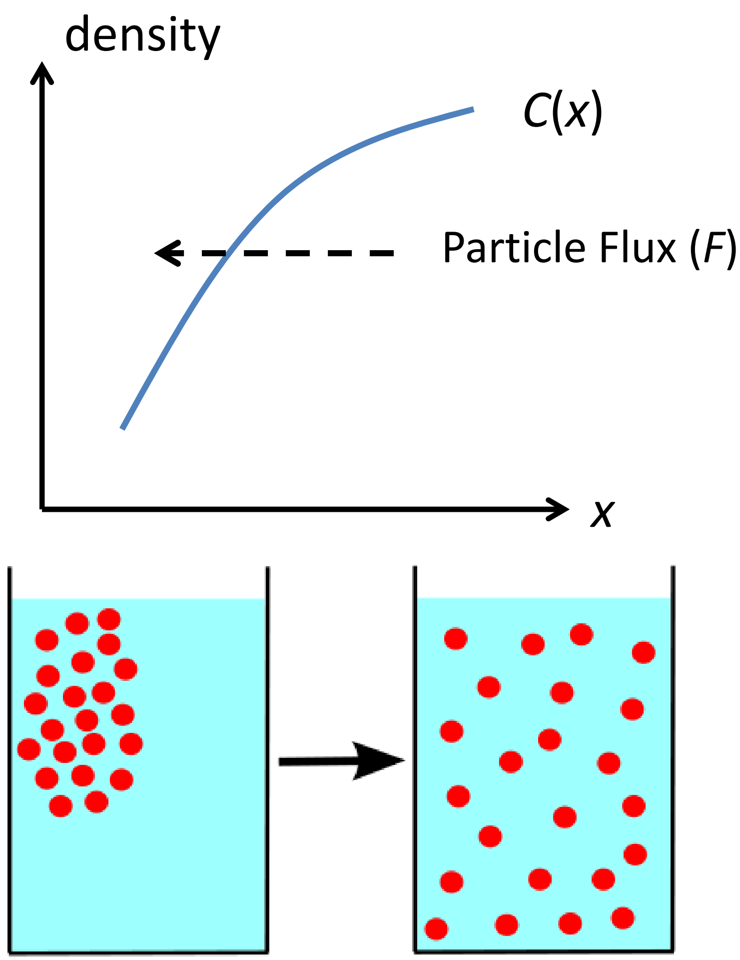
Holes: $j_p^{\text{drift}} = qpv_p = qp(\mu_p E) = qp\mu_p E$

Total: $j^{\text{drift}} = j_n^{\text{drift}} + j_p^{\text{drift}} = \underbrace{q(n\mu_n + p\mu_p)}_{\sigma}E$

Up to now we assumed that *n*, *p* are constant over space

When a spatial gradients of *n* and/or *p* exists, diffusion currents arise

Diffusion



- Fick's Law of diffusion:

$$F = -D\frac{dC}{dx}$$

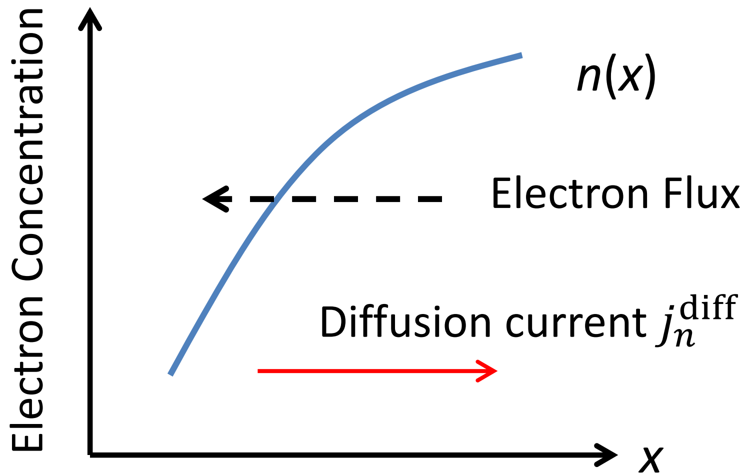
F: Particle flux (amount per unit area per unit time) [cm^{-2}/s]

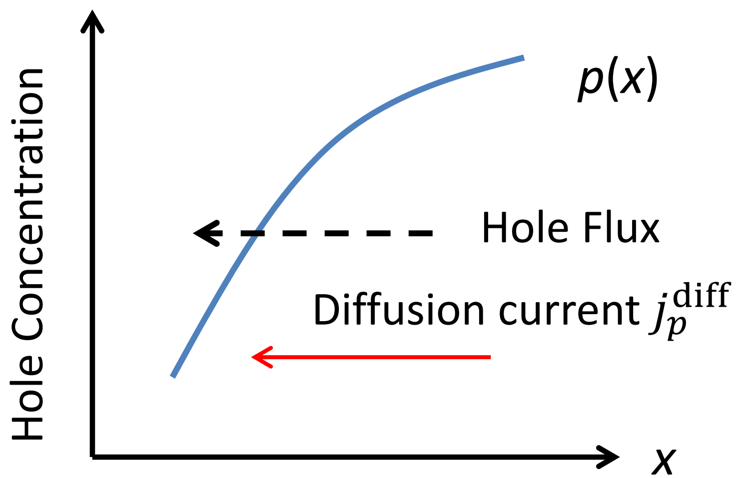
D: Diffusion coefficient or diffusivity [cm^2/s]

C: Concentration (density) of particles [cm^{-3}]

Flux direction opposite to gradient

Conduction Mechanisms – Diffusion Current





Electron flux: $F_n = -D_n \frac{dn}{dx}$

Electron diffusion current:

$$j_n^{\mathrm{diff}} = (-q)F_n = qD_n \frac{dn}{dx}$$

Hole flux: $F_p = -D_p \frac{dp}{dx}$

Hole diffusion current:

$$j_p^{\mathrm{diff}} = (+q)F_n = -qD_p \frac{dp}{dx}$$

Notice the sign difference!

Charge Transport in Semiconductors

- Total current = drift + diffusion

$$j_n^{\text{tot}} = j_n^{\text{drift}} + j_n^{\text{diff}} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$j_p^{\text{tot}} = j_p^{\text{drift}} + j_p^{\text{diff}} = qp\mu_p E - qD_p \frac{dp}{dx}$$

- 3-dimensional case (vector notation):

$$\mathbf{j}_n^{\text{tot}} = qn\mu_n \mathbf{E} + qD_n \nabla n$$

$$\mathbf{j}_p^{\text{tot}} = qp\mu_p \mathbf{E} - qD_p \nabla p$$

Einstein Relationship

- D_n, D_p: diffusivity of electrons and holes
- μ_n, μ_p : mobility of electrons and holes
- Einstein's relationship:

$$\frac{D}{\mu} = \frac{kT}{q}$$

Rhymes both forwards and backwards!

- *kT*/*q* is called "thermal voltage" (V_T)
- At 300 K, $\frac{kT}{q} = \frac{8.62\times10^{-5}\mathrm{eV/K}\times300\mathrm{K}}{1e} = 25.9\mathrm{mV}$

Gauss's Law and Poisson's Equation

- Electric field is related to the charge density

$$\boldsymbol{\nabla} \cdot \epsilon \mathbf{E} = \rho \quad \Rightarrow \quad \frac{dE}{dx} = \frac{\rho}{\epsilon} \text{ for 1D, } \epsilon \text{ constant}$$

- Poisson's Equation for electrostatics ($\mathbf{E} = -\nabla \varphi$)

$$\nabla^2 \varphi = -\frac{\rho}{\epsilon} \quad \Rightarrow \quad \frac{d^2 \varphi}{dx^2} = -\frac{\rho}{\epsilon}$$

- ρ: charge density [C/cm^3]
- φ: electrostatic potential [V]
- ϵ: permittivity [F/cm], $\epsilon = \epsilon_0 \epsilon_r$
  - ϵ_0: free space permittivity, 8.854 x 10^{-14} F/cm
  - ϵ_r: relative permittivity of the material (dimensionless)
    For Si, ϵ_r = 11.7; for SiO_2, ϵ_r = 3.9.

Vector Operators

- Real devices are 3D; 2/3D structures matter.
- Use vector operators:

$$\mathbf{E} = (E_x, E_y, E_z), \mathbf{J} = (J_x, J_y, J_z)$$

Gradient: $$\boldsymbol{\nabla} C = \left(\frac{\partial C}{\partial x}, \frac{\partial C}{\partial y}, \frac{\partial C}{\partial z}\right)$$

Divergence: $$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

Laplacian: $$\nabla^2 \varphi = \frac{d^2 \varphi}{dx^2} + \frac{d^2 \varphi}{dy^2} + \frac{d^2 \varphi}{dz^2}$$

Drift Diffusion Equations

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho(\boldsymbol{x})}{\varepsilon} \quad \Rightarrow \quad \frac{dE}{dx} = \frac{\rho(x)}{\varepsilon} \text{ for 1D}$$

$$\nabla^2 \varphi = -\frac{\rho(\boldsymbol{x})}{\varepsilon} \Rightarrow \quad \frac{d^2\varphi}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

$$\mathbf{j}_n^{\text{tot}} = qn\mu_n \mathbf{E} + qD_n \boldsymbol{\nabla} n \Rightarrow \; qn\mu_n E + qD_n \frac{dn}{dx}$$

$$\mathbf{j}_p^{\text{tot}} = qp\mu_p \mathbf{E} - qD_p \boldsymbol{\nabla} p \Rightarrow \; qp\mu_p E - qD_p \frac{dp}{dx}$$

$$\frac{\partial n}{\partial t} = -\, \boldsymbol{\nabla} \cdot \mathbf{j}_n^{\text{tot}} - \text{R} + \text{G} \qquad \text{(G is generation)}$$

$$\frac{\partial p}{\partial t} = -\, \boldsymbol{\nabla} \cdot \mathbf{j}_p^{\text{tot}} - \text{R} + \text{G} \qquad \text{(R is recombination)}$$

Device Analysis and Modeling

- The drift-diffusion equations are the basis for most analysis and design of semiconductor devices.